NATIONAL INSTITUTE OF TECHNOLOGY DURGAPUR

DEPARTMENT OF MATHEMATICS

Syllabus for PhD Admission Test

Algebra: Sets and equivalence relations and partitions, Permutations, combinations, pigeon-hole principle, inclusion-exclusion principle, derangements. Division algorithm for integers, unique factorizations, fundamental theorem of arithmetic, divisibility in Z, congruences, Chinese Remainder Theorem, Euler ϕ-function, primitive roots.
Groups: Cyclic groups, Permutation groups, Isomorphism of groups, Cosets and Lagrange's Theorem, Normal subgroups, Quotient groups, Group Homomorphisms, Cayley's Theorem, Group Action, Cauchy's Theorem, Sylow Theorems.
Rings, ideals, prime and maximal ideals, quotient rings, unique factorization domain, principal ideal domain, Euclidean domain. Polynomial rings and irreducibility criteria.
Fields, finite fields, field extensions, Galois Theory.

Complex Analysis: Algebra of complex numbers, the complex plane, polynomials, power series, transcendental functions such as exponential, trigonometric and hyperbolic functions. Analytic functions, Cauchy-Riemann equations. Contour integral, Cauchy's theorem, Cauchy's integral formula, Liouville's theorem, Maximum modulus principle, Schwarz lemma, Open mapping theorem. Taylor series, Laurent series, calculus of residues. Conformal mappings, Mobius transformations.

Linear Algebra: Vector spaces, subspaces, linear dependence, basis, dimension, algebra of linear transformations. Algebra of matrices, rank and determinant of matrices, linear equations. Eigenvalues and eigenvectors, Cayley-Hamilton theorem. Matrix representation of linear transformations. Change of basis, canonical forms, diagonal forms, triangular forms, Jordan blocks and Jordan forms, rational canonical form.
Inner product spaces, orthonormal basis. Quadratic forms, reduction and classification of quadratic forms.

Ordinary Differential Equations (ODEs): Existence and uniqueness of solution of initial value problems, singular solutions of first order ODEs, system of first order ODEs. Series solution around ordinary point and a regular singular point, Bessel functions and Legendre polynomials. Variation of parameter, Green's function, Sturm Liouville problems, Perturbation theory for two-dimensional linear system.

Partial Differential Equations (PDEs): Lagrange and Charpit methods for solving first order PDEs, Cauchy problem for first order PDEs. Classification of second order PDEs, Interior and exterior Dirichlet problems in polar coordinates; General solution of higher order PDEs with constant coefficients, Method of separation of variables for Laplace, Heat and Wave equations. Green's function. Fourier series and Fourier transform and Laplace transform methods of solutions for the above equations.

Real Analysis: Elementary set theory, finite, countable and uncountable sets, Real number system as a complete ordered field, Archimedean property, supremum, infimum. Sequences and series, convergence, limsup, liminf. Bolzano Weierstrass theorem. Continuity, uniform continuity, differentiability, mean value theorem. Sequences and series of functions, uniform convergence. Riemann sums and Riemann integral, Improper Integrals.
Monotonic functions, types of discontinuity, functions of bounded variation, Lebesgue Measure: Borel sets. outer measure and measurable sets, construction of a nonmeasurable set, Lebesgue measure on R^n, measure space, measurable functions.
Lebesgue integral. monotone convergence theorem and fatou's Lemma, properties of integrable functions and dominated convergence theorem.
Riemann Integral: Riemann integral and its properties, characterization of Riemann integrable functions. Riemann-Lebesgue lemma, Drawbacks of Riemann Integral, Lebesgue's recipe, Riemann-Stieltjes integral.
Functions of several variables, directional derivative, partial derivative, derivative as a linear transformation, inverse and implicit function theorems.
Metric spaces, compactness, connectedness. Normed linear Spaces. Spaces of continuous functions as examples.

Functional Analysis: Metric space: Continuity and equivalent metrics, compactness, Cauchy sequences, completeness and completion of metric space. Banach spaces and fundamental theorems: normed linear spaces, Banach spaces, equivalent norms, finite dimensional normed linear spaces, Riesz lemma. bounded linear transformations. uniform boundedness theorem, open mapping theorem, closed graph theorem, linear functionals, Hahn-Banach theorem, Dual space. Hilbert spaces: Real inner product spaces and its complexification. Orthogonal complement and projection theorem. Operators: Riesz representation theorem, adjoint of an operator on a Hilbert space, reflexivity of Hilbert spaces, self-adjoint operators, positive operators, projection operators.

Topology: Topological spaces, basis, sense sets, subspace, product topology, Continuous Functions, homeomorphisms, product and box topology. Metric topology, quotient topology. connected and path connected spaces, connected sets in real line, components, local connectedness, compact spaces, compact sets in real line, Heine-Borel theorem, connectedness and compactness, countability axioms, separation axioms.

Vector Analysis: Divergence and curl of a vector point function – solenoidal and irrational functions – physical interpretation of divergence and curl of a vector point function. Integration of vector functions – Line, surface and volume integrals. Guass - Divergence Theorem – Green's Theorem – Stoke's Theorem

Probability and statistics: Sample space, discrete probability, independent events, Bayes theorem. Random variables and distribution functions (univariate and multivariate); expectation and moments. Independent random variables, marginal and conditional distributions. Characteristic functions. Probability inequalities (Tchebyshef, Markov, Jensen). Modes of convergence, weak and strong laws of large numbers, Central Limit theorems.
Markov chains with finite and countable state space, classification of states, limiting behaviour of n-step transition probabilities, stationary distribution, Poisson and birth-and-death processes.
Standard discrete and continuous univariate distributions. sampling distributions, standard errors and asymptotic distributions, distribution of order statistics and range.

Methods of estimation, properties of estimators, confidence intervals. Tests of hypotheses: most powerful and uniformly most powerful tests, likelihood ratio tests. Analysis of discrete data and chi-square test of goodness of fit. Large sample tests.

Integral Transforms: Fourier Integrals and Fourier Transforms. Inverse of Fourier Transform, Sine and Cosine transforms. Perseval's Relations. Application to ODE & PDE. Laplace Transforms: Laplace Transform of Derivatives, Laplace Transform of Integrals, Inverse of Laplace Transform, Application to ODE.

Linear Integral Equations: Linear integral equation of the first and second kind of Fredholm and Volterra type, Solutions with separable kernels. Characteristic numbers and eigenfunctions, resolvent kernel.

Mechanics: Generalized coordinates, Lagrange's equations, Hamilton's canonical equations, Hamilton's principle and principle of least action, Two-dimensional motion of rigid bodies, Euler's dynamical equations for the motion of a rigid body about an axis, theory of small oscillations. Legendre transformation, Canonical Transformation, Generating function, Poisson bracket, Identities on Poisson brackets. Variation of a functional, Euler-Lagrange equation, Necessary and sufficient conditions for extrema. Variational methods for boundary value problems in ordinary and partial differential equations.

Numerical Analysis: Interpolation, Central difference interpolation, spline and Hermite interpolations. Numerical solutions of algebraic equations, Method of iteration and Newton-Raphson method, Rate of convergence. Numerical Integration: Newton-Cote's and Gaussian quadrature. Solution of a system of linear equations by Gauss-elimination, LU-decomposition and Gauss-Seidel methods, Ill-conditioned linear systems. Solution of ordinary differential equation, Picard method, Single step and multi-step methods, Predictor-corrector method. Stability analysis.

Operations Research: Linear programming problems, optimal solution, extreme points, non-linear programming problem. Dual problem and duality theorems, dual simplex method and its application in post optimality analysis. Balanced and unbalanced transportation problems, Vogel's approximation method for solving transportation problems. Travelling salesman problem. Hungarian method for solving assignment problems. Sensitivity analysis. Integer programming problems: cutting plane and branch and bound methods. Game theory, Probabilistic game, Reduction of a game problem to linear programming problem. Bimatrix games.
Elementary queuing and inventory models. Steady-state solutions of Markovian queuing models: M/M/1, M/M/1 with limited waiting space, M/M/C, M/M/C with limited waiting space.

Graph Theory: Directed graphs and weighted graphs, Types of graphs, Graph isomorphism, Sum and product of graphs, Components, Connected and disconnected graphs. Trees, Binary tree, Planar and non-planar graphs, Kuratowaski's graphs, Homeomorphic graphs, Geometric and combinatorial duals. Colouring, chromatic polynomial. Matching. Matrix representation of graphs. Spanning tree and algorithms, Shortest path algorithms.

Computer Programing and Data Structure: Concept of computer programming, C-programming language, loops, files, arrays, pointers. Stack, linked list, tree, time complexity of algorithm, sorting and searching algorithms, binary tree.